International Journal of Mathematics and Physical Sciences Research ISSN 2348-5736 (Online)
Vol. 2, Issue 2, pp: (164-167), Month: October 2014 – March 2015, Available at: www.researchpublish.com

fgb-Connectedness in Fine- Topological Spaces

Rajak K.

Assistant Professor, Department of Mathematics, St. Aloysius College (Autonomous), Jabalpur, M. P., India

***Abstract*: Powar P. L. and Rajak K. have introduced fine-topological space which is a special case of generalized topological space. In this paper, we have introduced fb-open sets, fgb-open sets and define some new continuous functions. Also, we have introduced fgb-connectedness and their properties are studied.**

Mathematics Subject Classification: 54XX, 54CXX.

***Keywords*: fgb-open set, fgb-closed sets, fgb-compact spaces, fgb-complete accumulation point, fgb-connectedness, fine-open sets, fine-closed sets.**

1. INTRODUCTION

Many researchers have investigated the basic properties of connectedness. The productivity and fruitfulness of these notions of connectedness motivated mathematicians to generalize these notions. In the course of these attempts many stronger and weaker forms of and connectedness have been introduced and investigated.

D. Andrijevic [1] introduced a new class of generalized open sets in a topological space called b-open sets. The class of b-open sets generates the same topology as the class of pre-open sets. Since the advent of this notion, several research paper with interesting results in different respects came into existence (See [3, 4, 5, 7, 8, 9]). M. Ganster and M. Steiner [6] introduced and studied the properties of gb-closed sets in topological spaces.

Powar P. L. and Rajak K. [14], have investigated a special case of generalized topological space called fine topological space. In this space, the authors have defined a new class of open sets namely fine-open sets which contains all $\alpha-$open sets, $\beta-$open sets, semi-open sets, pre-open sets, regular open sets etc. By using these fine-open sets they have defined fine-irresolute mappings which includes pre-continuous functions, semi-continuous function, $\alpha-$continuouos function, $\beta-$continuous functions, $\alpha-$irresolute functions, $\beta-$irresolute functions, etc (cf. [10]-[14]).

The aim of this paper, we have introduced fb-open sets, fgb-open sets and define some new continuous functions. Also, we have introduced fgb-connectedness and their properties are studied.

2. PRELIMINARIES

Throughout this paper (X, τ), (Y, σ) are topological spaces with no separation axioms assumed unless otherwise stated. Let $A \subseteq X$. The closure of A and the interior of A will be denoted by Cl(A) and Int(A) respectively.

Definition 2.1 A subset A of X is said to be b-open [1] if $A \subseteq Int(Cl(A)) \cup Cl(Int(A))$. The complement of b-open set is said to be b-closed. The family of all b-open sets (respectively b-closed sets) of (X, τ) is denoted by $bO(X, \tau)$ [respectively $bCL(X, \tau)$].

Definition 2.2 Let A be a subset of X. Then

(i) b-interior [1] of A is the union of all b-open sets contained in A.

(ii) b-closure [1] of A is the intersection of all b-closed sets containing A.

The b-interior [respectively b-closure] of A is denoted by bInt(A) [respectively bCl(A)].

Definition 2.3 Let A be a subset of X. Then A is said to be gb-closed [5] if $bCL(A) \subseteq U$ whenever $A \subseteq U$ and $U \in bO(X, \tau)$. The complement of gb-closed [6] set is called gb-open. The family of all gb-open [respectively gb-closed] sets of (X, τ) is denoted by $gbO(X, \tau)$ [respectively $gbCL(X, \tau)$].

Definition 2.4 The gb-closure [6] of a set A is the intersection of all gb-closed sets containing A and is denoted by gbCl(A).

Definition 2.5 The gb-interior [6] of a set A is the union of all gb-open sets contained in A and is denoted by gbInt(A).

Remark 2.6 Every b-closed set is gb-closed.

Definition 2.7 A function f: X → Y is said to be gb-continuous [5] if $f-1(V)$ is gb-closed in X for every closed set V of Y (c.f. [2]).

Definition 2.8 A function f : X → Y is said to be gb-irresolute [5] if $f-1(V)$ is gb-closed in X for every gb-closed set V of Y (c.f. [2]).

Definition 2.9 A topological space X is said to be gb-connected if X cannot be expressed as a disjoint union of two non-empty gb-open sets. A subset of X is gb-connected if it is gb-connected as a subspace (c.f. [2]).

Definition 2.10 A topological space X is said to be Tgb-space if every gb-closed subset of X is closed subset of X (c.f. [2]).

Definition 2.11 Let (X, τ) be a topological space we define

τ (Aα) = τα (say) = {Gα(≠X) : Gα ∩ Aα= ϕ, for Aα ∈ τ and Aα≠ ϕ,X, for some α ∈ J, where J is the index set.}

Now, we define τf = {ϕ,X,∪ α∈J {τα}}

The above collection τf of subsets of X is called the fine collection of subsets of X and (X, τ, τf) is said to be the fine space X generated by the topology τ on X (cf. [14]).

Definition 2.12 A subset U of a fine space X is said to be a fine-open set of X, if U belongs to the collection τf and the complement of every fine-open sets of X is called the fine-closed sets of X and we denote the collection by Ff (cf. [14]).

Definition 2.13 Let A be a subset of a fine space X, we say that a point x ∈ X is a fine limit point of A if every fine-open set of X containing x must contains at least one point of A other than x (cf. [14]).

Definition 2.14 Let A be the subset of a fine space X, the fine interior of A is defined as the union of all fine-open sets contained in the set A i.e. the largest fine-open set contained in the set A and is denoted by fInt(cf. [14]).

Definition 2.15 Let A be the subset of a fine space X, the fine closure of A is defined as the intersection of all fine-closed sets containing the set A i.e. the smallest fine-closed set containing the set A and is denoted by fcl(cf. [14]).

Definition 2.16 A function f : (X, τ, τf) → (Y, τ ′, τf′) is called fine-irresolute (or f-irresolute) if f−1(V) is fine-open in X for every fine-open set V of Y (cf. [14]).

3. fgb-CONNECTEDNESS

In this section we have defined fgb-connectedness in fine-topological space.

Definition 3.1 A subset A of X is said to be fb-open if $A \subseteq fInt(fcl(A)) \cup fcl\,(fInt(A))$. The complement of fb-open set is said to be fb-closed. The family of all fb-open sets (respectively fb-closed sets) of (X, τ) is denoted by fbO(X, τ) [respectively fbCL(X, τ)].

Example 3.2 Let X = {a, b, c} with the topology τ = {∅, X, {a, b}}, τf = {∅, X, {a}, {a, b}, {a, c}, {b}, {b, c}}, Ff = {∅, X, {b, c}, {c}, {b}, {a, c}, {a}}. It may be easily check that the only fb-open sets are $\phi, X, a, \{b\}, \{a,b\}, \{a,c\}, \{b,c\}$.

Remark 3.3 Every b-open set is fb-open and every b-open and fb-open set is fine-open.

Definition 3.4 Let A be a subset of X. Then

(i) fb-interior of A is the union of all fb-open sets contained in A.

(ii) fb-closure of A is the intersection of all b-closed sets containing A.

The fb-interior [respectively fb-closure] of A is denoted by $fbInt(A)$ [respectively$fbcl(A)$].

Example 3.5 Let X = {a, b, c} with the topology τ = {∅, X, {a, b}}, τf = {∅, X, {a}, {a, b}, {a, c}, {b}, {b, c}}, Ff = {∅, X, {b, c}, {c}, {b}, {a, c}, {a}}. Let $S=\{a,b\}$ the fb-interior of S is {a, b} and the fb-closure of S is X.

Definition 3.6 Let A be a subset of X. Then A is said to be fgb-closed if $fbInt(A) \subseteq$ U whenever A ⊆ U and U ∈ fbO(X, τ). The complement of fgb-closed set is called fgb-open. The family of all fgb-open [respectively fgb-closed] sets of (X, τ) is denoted by fgbO(X, τ) [respectively fgbCL(X, τ)].

Example 3.7 Let X = {a, b, c} with the topology τ = { ∅, X, {a}, {a, b}}, τf = {∅, X, {a}, {a, b}, {a, c}, {b}, {b, c}}, Ff = {∅, X, {b, c}, {c}, {b}, {a, c}, {a}}. It can be easily check that the only fgb-open sets are $\phi, X\ a\ ,\ b\ ,\ a,b\ ,\ a,c\ ,\ b,c$.

Remark 3.8 Every fb-open set is fgb-open.

Definition 3.9 The fgb-closure of a set A is the intersection of all $fgb-$closed sets containing A and is denoted by$fgbcl(A)$.

Definition 3.10 The fgb-interior of a set A is the union of all fgb-open sets contained in A and is denoted by $fgbInt\ A$.

Example 3.11 Let X = {a, b, c} with the topology τ = { ∅, X, {a}}, τf = {∅, X, {a}, {a, b}, {a, c}}, Ff = {∅, X, {b, c}, {c}, {b}}. Let S={ b} the fgb-interior of S is {b} and the fgb-closure of S is {b}.

Definition 3.12 A function f: X → Y is said to be fgb-continuous if $f-1(V\ )$ is fgb-closed in X for every fine-closed set V of Y.

Example 3.13 Let X = {a, b, c} with the topology $\tau= \phi,X,\ a,b\ ,\ b\ ,\ b,c\ ,\tau f= \phi,X,\ a\ ,\ a,b\ ,\ a,c\ ,\ b\ ,\ b,c\ ,\ c\ ,\ Ff=\{\phi\ X,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\}\ \}$ and let Y = {1, 2, 3} with the topology $\tau'= \phi,Y,$ 1 , 1,2 , τf'={ϕ, Y, {1}, {1, 2}, {1, 3}, {2}, {2, 3} }. We define a mapping $f{:}X{\to}Y$ by f(a) = 1, f(b) = 2, f(c) = 3. It may be easily check that the only fgb-open sets of Y are $\phi,Y,$ 1 , 1,2 , 1,3 , 2 , 2,3 whose pre-images are $\phi,X,\ a\ ,\ a,b\ ,\ a,c\ ,\ b\ ,\ b,c$ which are fine-closed in X. Hence, f is fgb-continuous.

Definition 3.14 A function f: X → Y is said to be fgb-irresolute if $f-1(V\ )$ is fgb-closed in X for every fgb-closed set V of Y.

Example 3.15 Let X = {a, b, c} with the topology $\tau= \phi,X,\ a,b\ ,\tau f=\{\phi,X,\ a\ ,\ a,b\ ,\ a,c\ ,\ b\ ,\ b,c\ \}$ and let Y = {1, 2, 3} with the topology $\tau'= \phi,Y,$ 1 , 1,2 ,$\tau f'= \phi,Y,$ 1 , 1,2 , 1,3 , 2 , 2,3 . We define a mapping $f{:}X{\to}Y$ by f(a) = 1, f(b) = 2, f(c) = 3. It may be easily checked that the only fgb-open sets of Y are $\phi,Y,$ 1 , 1,2 , 1,3 , 2 , 2,3 whose pre-images are $\phi,X,\ a\ ,\ a,b\ ,\ a,c\ ,\ b\ ,\ b,c$ which are fgb-open in X. Hence, f is fgb-irresolute.

Definition 3.16 A topological space X is said to be fgb-connected if X cannot be expressed as a disjoint union of two non-empty fgb-open sets. A subset of X is fgb-connected if it is fgb-connected as a subspace.

Example 3.17 Let X = {a, b} and let τ = {X, υ, {a}}, $\tau f=\{\phi,X,\ a\ ,\ a,b\ ,\ a,c\ \}$. Then it is fgb-connected space.

Definition 3.18 A topological space X is said to be fTgb-space if every fgb-closed subset of X is fine-closed subset of X.

Example 3.19 Let X = {a, b} and let τ = {X, υ, {a, b}, {b}, {b, c}}, τf={ϕ, X, {a}, {a, b}, {a, c}, {b}, {b, c}, {c}}. Then it is gb-connected.

Theorem 3.20 If f : X → Y is a fgb-continuous and X is fgb-connected, then Y is fine-connected.

Proof: Suppose that Y is not fine-connected. Let Y = A∪B where A and B are disjoint non-empty fine-open set in Y. Since f is fgb-continuous and onto, $X = f-1(A) \cup f-1(B)$ where $f-1(A)$ and $f-1(B)$ are disjoint non-empty fgb-open sets in X. This contradicts the fact that X is fgb-connected. Hence, Y fgb-connectedness in topological spaces.

Theorem 3.21 If f : X → Y is a fgb-irresolute surjection and X is fgb-connected, then Y is fgb-connected.

Proof: Suppose that Y is not fgb-connected. Let Y = A ∪ B where A and B are disjoint non-empty fgb-open set in Y. Since f is fgb-irresolute and onto, $X = f-1(A) \cup f-1(B)$ where $f-1(A)$ and $f-1(B)$ are disjoint non-empty fgb-open sets in X. This contradicts the fact that X is fgb-connected. Hence Y is fine-connected.

Theorem 3.22 In a fine-topological space (X, τ, τf) with at least two points, if fbO(X, τ, τf) = fbCL(X, τ, τf) then X is not fgb-connected.

Proof: By hypothesis we have fbO(X, τ, τf)) = fbCL(X, τ, τf)) and every fb-closed set is fgb-closed, there exists some non-empty proper subset of X which is both fgb-open and fgb-closed in X. So by last Theorem 4.7 we have X is not fgb-connected.

Theorem 3.23 Suppose that X is a fTgb-space then X is fine-connected if and only if it is fgb-connected.

Proof: Suppose that X is fine-connected. Then X cannot be expressed as disjoint union of two non-empty proper subsets of X. Suppose X is not a fgb-connected space. Let A and B be any two fgb-open subsets of X such that X = A∪B, where A ∩ B = υ and A ⊂ X, B ⊂ X. Since X is fTgb-space and A,B are fgb-open, A,B are open subsets of X, which contradicts that X is fine-connected. Therefore X is fgb-connected. Conversely, every fine-open set is fgb-open. Therefore every fgb-connected space is fine-connected.

Theorem 3.24 If the fgb-open sets C and D form a separation of X and if Y is fgb-connected subspace of X, then Y lies entirely within C or D.

Proof: Since C and D are both fgb-open in X the sets C ∩ Y and D ∩ Y are fgb-open in Y these two sets are disjoint and their union is Y. If they were both non-empty, they would constitute a separation of Y. Therefore, one of them is empty. Hence Y must lie entirely in C or in D.

Theorem 3.25 Let A be a fgb-connected subspace of X. If A ⊂ B ⊂ fgbClA then B is also fgb-connected.

Proof: Let A be fgb-connected and let A ⊂ B ⊂ gbCl(A). Suppose that B = C∪D is a separation of B by fgb-open sets. Then by Theorem 4.11 above A must lie entirely in C or in D. Suppose that A ⊂ C, then fgbCl(A) ⊆ fgbCl(C). Since fgbCl(C) and D are disjoint, B cannot intersect D. This contradicts the fact that D is non-empty subset of B. So D = υ which implies B is fgb-connected.

REFERENCES

[1] Andrijevic D.,On b-open sets, Math. Vesnik, 48(1996), No. 1-2, 59-64.

[2] Benchali S.S. and Bansali P. M., gb-Compactness and gb-connectedness in Topological Space, Int. J. Contemp. Math. Sciences, 6(2011), No. 10, 465-475.

[3] Caldas M. and Jafari S.,On some applications of b-open sets in topological spaces, Kochi, J. Math., 2(2007), 11-19.

[4] Ekici E. and Caldas M., Slightly γ-continuous functions, Bol.Soc. Parana. Mat.,(3) 22(2004), No. 2, 63-74.

[5] Ganster M. and Steiner M., On some questions about b-open sets,Questions - Answers Gen. Topology,25(2007), No. 1, 45-52.

[6] Ganster M. and Steiner M., On bτ-closed sets, Appl. Gen.Topol., 8 (2007), No. 2, 243-247.

[7] Nasef A. A., Some properties of contra-γ-continuous functions, Chaos Solitons Fractals, 24 (2005), No. 2, 471-477.

[8] Nasef A. A., On b-locally closed sets and related topics, Chaos Solitons Fractals, 12 (2001), No. 10, 1909-1915.

[9] Park J. H., θ-b-continuous functions, Acta Math. Hungar, 110(2006), No. 4, 347-359. Received: August, 2010

[10] Powar P. L. and Rajak K., Some new concept of Continuity in Generalized Topological Space, International Journal of Computer Application, 38 (2012), No. 5,12-17.

[11] Powar P. L. and Rajak K., Applications of α and β-irresolute functions, International Journal for Electro Computational World Knowledge Interface, 1(2012), No. 5, 23-29.

[12] Powar P. L. and Rajak K., Ultra Separation Axioms in Generalized Topological Space, International Journal for Electro Computational World Knowledge Interface, 1(2011), No. 4, 34-40.

[13] Powar P. L. and Rajak K., Study of g-α-irresolute functions in the special class of Generalized Topological Space, International Journal of Computer Application, 31(2011), 1-5,

[14] Powar P. L. and Rajak K., Fine irresolute mappings, Journal of Advance Studies in Topology, 3 (2012), No. 4, , 125-139.